

Calabi–Yau Phases of the Superstring Vacuum

Yuri M. Malyuta

*Institute for Nuclear Research, National Academy of Sciences of Ukraine
252028 Kiev, Ukraine*

Nikolay N. Aksenov

*Space Research Institute, National Academy of Sciences of Ukraine
252022 Kiev, Ukraine*

E-mail: aks@d310.icyb.kiev.ua

ABSTRACT

It is shown that the model $X_{14}(7, 3, 2, 1, 1)$ has two Calabi–Yau phases.

1 Introduction

It was discovered by Berglund, Katz and Klemm [1] that the model $X_9(3, 2, 2, 1, 1)$ has two Calabi–Yau phases. Recently this type of models has been the focus of work in the context of the heterotic/type II string duality [2, 3].

In this work we show that the model $X_{14}(7, 3, 2, 1, 1)$ also has two Calabi–Yau phases.

2 Phase I

Following the prescriptions of the papers [4, 5] we can derive the generators of the Mori cone, the principal parts of the Picard–Fuchs operators and the normalization.

The generators of the Mori cone are

$$\begin{aligned} l^{(1)} &= (0; 7, 0, 1, -4, -2, -2), \\ l^{(2)} &= (-2; -3, 1, 0, 2, 1, 1). \end{aligned}$$

The principal parts of the Picard–Fuchs operators are

$$\begin{aligned} L_1 &= (7\theta_1 - 3\theta_2)\theta_1, \\ L_2 &= (\theta_2 - 2\theta_1)^3. \end{aligned}$$

The normalization is $K_{111}^0 = 9$.

With the computer program INSTANTON [5] we can obtain the Yukawa couplings and the instanton numbers. The Yukawa couplings are

$$\begin{aligned} K_{111} &= 220 q_1 q_2^2 - 440 q_1 q_2^3 + 1100 q_1 q_2^4 + 2300 q_1^2 q_2^4 \\ &\quad - 7040 q_1 q_2^5 + 184400 q_1^2 q_2^5 + 62920 q_1 q_2^6 - 742200 q_1^2 q_2^6 \\ &\quad + 6160 q_1^3 q_2^6 - 668360 q_1 q_2^7 + 6582800 q_1^2 q_2^7 + 407048112 q_1^3 q_2^7 \\ &\quad + 7891400 q_1 q_2^8 - 75772500 q_1^2 q_2^8 - 100073600 q_1 q_2^9 + \dots, \\ K_{112} &= 440 q_1 q_2^2 - 1320 q_1 q_2^3 + 4400 q_1 q_2^4 + 4600 q_1^2 q_2^4 \\ &\quad - 35200 q_1 q_2^5 + 461000 q_1^2 q_2^5 + 377520 q_1 q_2^6 - 2226600 q_1^2 q_2^6 \\ &\quad + 12320 q_1^3 q_2^6 - 4678520 q_1 q_2^7 + 23039800 q_1^2 q_2^7 + 949778928 q_1^3 q_2^7 \\ &\quad + 63131200 q_1 q_2^8 - 303090000 q_1^2 q_2^8 - 900662400 q_1 q_2^9 + \dots, \end{aligned}$$

$$\begin{aligned}
K_{122} = & 880 q_1 q_2^2 - 3960 q_1 q_2^3 + 17600 q_1 q_2^4 + 9200 q_1^2 q_2^4 - 176000 q_1 q_2^5 \\
& + 1152500 q_1^2 q_2^5 + 2265120 q_1 q_2^6 - 6679800 q_1^2 q_2^6 + 24640 q_1^3 q_2^6 \\
& - 32749640 q_1 q_2^7 + 80639300 q_1^2 q_2^7 + 2216150832 q_1^3 q_2^7 \\
& + 505049600 q_1 q_2^8 - 1212360000 q_1^2 q_2^8 - 8105961600 q_1 q_2^9 + \dots,
\end{aligned}$$

$$\begin{aligned}
K_{222} = & 3 q_2 - 45 q_2^2 + 1760 q_1 q_2^2 + 732 q_2^3 - 11880 q_1 q_2^3 - 12333 q_2^4 \\
& + 70400 q_1 q_2^4 + 18400 q_1^2 q_2^4 + 211878 q_2^5 - 880000 q_1 q_2^5 + 2881250 q_1^2 q_2^5 \\
& - 3685140 q_2^6 + 13590720 q_1 q_2^6 - 20039400 q_1^2 q_2^6 + 49280 q_1^3 q_2^6 \\
& + 64639725 q_2^7 - 229247480 q_1 q_2^7 + 282237550 q_1^2 q_2^7 + 5171018608 q_1^3 q_2^7 \\
& - 1140830253 q_2^8 + 4040396800 q_1 q_2^8 - 4849440000 q_1^2 q_2^8 + 20228948103 q_2^9 \\
& - 72953654400 q_1 q_2^9 - 360011938170 q_2^{10} + \dots.
\end{aligned}$$

The instanton numbers are recorded in Table 1.

Table 1

$n_{j,k}$	$j = 0$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$n_{j,0}$	0	0	0	0	0
$n_{j,1}$	3	0	0	0	0
$n_{j,2}$	-6	220	0	0	0
$n_{j,3}$	27	-440	0	0	0
$n_{j,4}$	-192	1100	260	0	0
$n_{j,5}$	1695	-7040	23050	0	0
$n_{j,6}$	-17064	62920	-92720	220	0
$n_{j,7}$	188454	-668360	822850	15075856	0
$n_{j,8}$	-2228160	7891400	-9471700	4571160	260

3 Phase II

The generators of the Mori cone are

$$\begin{aligned}
l^{(1)} &= (-4; 1, 2, 1, 0, 0, 0), \\
l^{(2)} &= (-2; -3, 1, 0, 2, 1, 1).
\end{aligned}$$

The principal parts of the Picard–Fuchs operators are

$$\begin{aligned}
L_1 &= (\theta_1 - 3\theta_2)\theta_1, \\
L_2 &= \theta_2^3.
\end{aligned}$$

The normalization is $K_{111}^0 = 9$.

Application of INSTANTON [5] gives the Yukawa couplings and the instanton numbers. The Yukawa couplings are

$$\begin{aligned}
K_{111} = & 220 q_1 + 2300 q_1^2 + 6160 q_1^3 + 18940 q_1^4 + 27720 q_1^5 + 64400 q_1^6 + 75680 q_1^7 \\
& + 152060 q_1^8 + 166540 q_1^9 + 289800 q_1^{10} - 440 q_1 q_2 + 184400 q_1^2 q_2 \\
& + 407048112 q_1^3 q_2 + 41624475840 q_1^4 q_2 + 1828147967000 q_1^5 q_2 \\
& + 48876855669360 q_1^6 q_2 + 928244560989200 q_1^7 q_2 + 13662697686429696 q_1^8 q_2 \\
& + 164713342061738520 q_1^9 q_2 + 1100 q_1 q_2^2 - 742200 q_1^2 q_2^2 + 123421320 q_1^3 q_2^2 \\
& - 92301136560 q_1^4 q_2^2 + 10878624450000 q_1^5 q_2^2 + 29170004481428400 q_1^6 q_2^2 \\
& + 4982997178421290560 q_1^7 q_2^2 + 400064510148214852800 q_1^8 q_2^2 - 7040 q_1 q_2^3 \\
& + 6582800 q_1^2 q_2^3 - 1562042240 q_1^3 q_2^3 + 383441785600 q_1^4 q_2^3 \\
& - 57413729228000 q_1^5 q_2^3 + 3167226854530640 q_1^6 q_2^3 \\
& - 10002877995518645440 q_1^7 q_2^3 + 62920 q_1 q_2^4 - 75772500 q_1^2 q_2^4 \\
& + 23668987320 q_1^3 q_2^4 - 4915958054200 q_1^4 q_2^4 + 799844578900500 q_1^5 q_2^4 \\
& - 79798035056700600 q_1^6 q_2^4 - 668360 q_1 q_2^5 + 986861200 q_1^2 q_2^5 \\
& - 383570510400 q_1^3 q_2^5 + 85307909769280 q_1^4 q_2^5 - 14459980360680440 q_1^5 q_2^5 \\
& + 7891400 q_1 q_2^6 - 13811657600 q_1^2 q_2^6 + 6429424311200 q_1^3 q_2^6 - \\
& - 1615989224562160 q_1^4 q_2^6 - 100073600 q_1 q_2^7 + 202613621200 q_1^2 q_2^7 \\
& - 109939230054720 q_1^3 q_2^7 + 1336128420 q_1 q_2^8 \\
& - 3072464694200 q_1^2 q_2^8 - 18546499400 q_1 q_2^9 + \dots ,
\end{aligned}$$

$$\begin{aligned}
K_{112} = & -440 q_1 q_2 + 92200 q_1^2 q_2 + 135682704 q_1^3 q_2 + 10406118960 q_1^4 q_2 \\
& + 365629593400 q_1^5 q_2 + 8146142611560 q_1^6 q_2 + 132606365855600 q_1^7 q_2 \\
& + 1707837210803712 q_1^8 q_2 + 18301482451304280 q_1^9 q_2 + 2200 q_1 q_2^2 \\
& - 742200 q_1^2 q_2^2 + 82280880 q_1^3 q_2^2 - 46150568280 q_1^4 q_2^2 \\
& + 4351449780000 q_1^5 q_2^2 + 9723334827142800 q_1^6 q_2^2 \\
& + 1423713479548940160 q_1^7 q_2^2 + 100016127537053713200 q_1^8 q_2^2 \\
& - 21120 q_1 q_2^3 + 9874200 q_1^2 q_2^3 - 1562042240 q_1^3 q_2^3 \\
& + 287581339200 q_1^4 q_2^3 - 34448237536800 q_1^5 q_2^3 + 1583613427265320 q_1^6 q_2^3 \\
& - 4286947712365133760 q_1^7 q_2^3 + 251680 q_1 q_2^4 - 151545000 q_1^2 q_2^4
\end{aligned}$$

$$\begin{aligned}
& +31558649760 q_1^3 q_2^4 - 4915958054200 q_1^4 q_2^4 + 639875663120400 q_1^5 q_2^4 \\
& -53198690037800400 q_1^6 q_2^4 - 3341800 q_1 q_2^5 + 2467153000 q_1^2 q_2^5 \\
& -639284184000 q_1^3 q_2^5 + 106634887211600 q_1^4 q_2^5 - 14459980360680440 q_1^5 q_2^5 \\
& +47348400 q_1 q_2^6 - 41434972800 q_1^2 q_2^6 + 12858848622400 q_1^3 q_2^6 \\
& -2423983836843240 q_1^4 q_2^6 - 700515200 q_1 q_2^7 + 709147674200 q_1^2 q_2^7 \\
& -256524870127680 q_1^3 q_2^7 + 10689027360 q_1 q_2^8 - 12289858776800 q_1^2 q_2^8 \\
& -166918494600 q_1 q_2^9 + \dots ,
\end{aligned}$$

$$\begin{aligned}
K_{122} = & -440 q_1 q_2 + 46100 q_1^2 q_2 + 45227568 q_1^3 q_2 + 2601529740 q_1^4 q_2 \\
& +73125918680 q_1^5 q_2 + 1357690435260 q_1^6 q_2 + 18943766550800 q_1^7 q_2 \\
& +213479651350464 q_1^8 q_2 + 2033498050144920 q_1^9 q_2 + 4400 q_1 q_2^2 \\
& -742200 q_1^2 q_2^2 + 54853920 q_1^3 q_2^2 - 23075284140 q_1^4 q_2^2 \\
& +1740579912000 q_1^5 q_2^2 + 3241111609047600 q_1^6 q_2^2 \\
& +406775279871125760 q_1^7 q_2^2 + 25004031884263428300 q_1^8 q_2^2 - 63360 q_1 q_2^3 \\
& +14811300 q_1^2 q_2^3 - 1562042240 q_1^3 q_2^3 + 215686004400 q_1^4 q_2^3 \\
& -20668942522080 q_1^5 q_2^3 + 791806713632660 q_1^6 q_2^3 \\
& -1837263305299343040 q_1^7 q_2^3 + 1006720 q_1 q_2^4 - 303090000 q_1^2 q_2^4 \\
& +42078199680 q_1^3 q_2^4 - 4915958054200 q_1^4 q_2^4 + 511900530496320 q_1^5 q_2^4 \\
& -35465793358533600 q_1^6 q_2^4 - 16709000 q_1 q_2^5 + 6167882500 q_1^2 q_2^5 \\
& -1065473640000 q_1^3 q_2^5 + 133293609014500 q_1^4 q_2^5 - 14459980360680440 q_1^5 q_2^5 \\
& +284090400 q_1 q_2^6 - 124304918400 q_1^2 q_2^6 + 25717697244800 q_1^3 q_2^6 \\
& -3635975755264860 q_1^4 q_2^6 - 4903606400 q_1 q_2^7 + 2482016859700 q_1^2 q_2^7 \\
& -598558030297920 q_1^3 q_2^7 + 85512218880 q_1 q_2^8 - 49159435107200 q_1^2 q_2^8 \\
& -1502266451400 q_1 q_2^9 + \dots ,
\end{aligned}$$

$$\begin{aligned}
K_{222} = & 3 q_2 - 440 q_1 q_2 + 23050 q_1^2 q_2 + 15075856 q_1^3 q_2 + 650382435 q_1^4 q_2 \\
& +14625183736 q_1^5 q_2 + 226281739210 q_1^6 q_2 + 2706252364400 q_1^7 q_2 \\
& +26684956418808 q_1^8 q_2 + 225944227793880 q_1^9 q_2 - 45 q_2^2 \\
& +8800 q_1 q_2^2 - 742200 q_1^2 q_2^2 + 36569280 q_1^3 q_2^2 - 11537642070 q_1^4 q_2^2 \\
& +696231964800 q_1^5 q_2^2 + 1080370536349200 q_1^6 q_2^2
\end{aligned}$$

$$\begin{aligned}
& +116221508534607360 q_1^7 q_2^2 + 6251007971065857075 q_1^8 q_2^2 + 732 q_2^3 \\
& -190080 q_1 q_2^3 + 22216950 q_1^2 q_2^3 - 1562042240 q_1^3 q_2^3 \\
& +161764503300 q_1^4 q_2^3 - 12401365513248 q_1^5 q_2^3 + 395903356816330 q_1^6 q_2^3 \\
& -787398559414004160 q_1^7 q_2^3 - 12333 q_2^4 + 4026880 q_1 q_2^4 - 606180000 q_1^2 q_2^4 \\
& +56104266240 q_1^3 q_2^4 - 4915958054200 q_1^4 q_2^4 + 409520424397056 q_1^5 q_2^4 \\
& -23643862239022400 q_1^6 q_2^4 + 211878 q_2^5 - 83545000 q_1 q_2^5 \\
& +15419706250 q_1^2 q_2^5 - 1775789400000 q_1^3 q_2^5 + 166617011268125 q_1^4 q_2^5 \\
& -14459980360680440 q_1^5 q_2^5 - 3685140 q_2^6 + 1704542400 q_1 q_2^6 \\
& -372914755200 q_1^2 q_2^6 + 51435394489600 q_1^3 q_2^6 - 5453963632897290 q_1^4 q_2^6 \\
& +64639725 q_2^7 - 34325244800 q_1 q_2^7 + 8687059008950 q_1^2 q_2^7 \\
& -1396635404028480 q_1^3 q_2^7 - 1140830253 q_2^8 + 684097751040 q_1 q_2^8 \\
& -196637740428800 q_1^2 q_2^8 + 20228948103 q_2^9 - 13520398062600 q_1 q_2^9 \\
& -360011938170 q_2^{10} + \dots .
\end{aligned}$$

The instanton numbers are recorded in Table 2.

Table 2

$n_{j,k}$	$j = 0$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$n_{j,0}$	0	220	260	220	260
$n_{j,1}$	3	-440	23050	15075856	650382435
$n_{j,2}$	-6	1100	-92720	4571160	-1442208140
$n_{j,3}$	27	-7040	822850	-57853400	5991277900
$n_{j,4}$	-192	62920	-9471700	876629160	-76811833000
$n_{j,5}$	1695	-668360	123357650	-14206315200	1332936090145
$n_{j,6}$	-17064	7891400	-1726456320	238126826300	-25249831736640
$n_{j,7}$	188454	-100073600	25326702650	-4071823335360	490864423233165
$n_{j,8}$	-2228160	1336128420	-384058094640	70536923244480	-9603462282665900

From Tables 1 and 2 we have the relation

$$n_{j,k+2j}(\text{phase I}) = n_{j,k}(\text{phase II}).$$

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